PHYSICAL REVIEW E VOLUME 52, NUMBER 2 AUGUST 1995

Experimental control of chaos in a periodically driven glow discharge

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(Received 27 March 1995)

The control of the chaotic state of ionization waves in the positive column of a periodically driven neon glow discharge is achieved by an active feedback technique based on the algorithm suggested by Ott, Grebogy, and Yorke [Phys. Rev. Lett. **64**, 1196 (1990)]. Unstable periodic orbits of both low and high periodicity could be stabilized by small, time-dependent variations of the modulation amplitude.

PACS number(s): 52.35.–g, 05.45.+b, 52.80.–s

Since Ott, Grebogy, and Yorke (OGY) presented their feedback algorithm to control the chaotic behavior of a deterministic system [1], many different examples for experimentally controlled systems have been reported in the literature (cf. Ref. [2] for an overview). The OGY idea relies on the key observation that a chaotic attractor typically has embedded within it a dense set of unstable periodic orbits (UPO's). One of these UPO's is chosen for stabilization through only small, time-dependent perturbations in an accessible control parameter. In the original OGY algorithm the control information is generated from the stability analysis of the local dynamics around the desired periodic orbit. This method essentially differs from the nonfeedback approaches to suppress (tame) chaos by perturbing a system either resonantly [3,4] or nonresonantly [5] with an external periodic signal. It has been shown that for particular systems only a small periodic perturbation is necessary to tame even deep chaotic states [6].

Clear experimental evidence for control of chaos has been demonstrated for a vast variety of different physical systems. Examples with a particular relationship to the present work are the periodically driven diode resonator [7–9], laser systems [10,11], and chemical waves [12]. The problem of controlling the chaotic state of a plasma, however, has attracted only little attention, even though the onset and control of chaos are of the greatest importance for the plasma turbulence pheomenon and the related fluctuation-induced transport. Some attempts to suppress plasma turbulence have been made by direct feedback of a fluctuating plasma parameter on the discharge [13]. The only report on experimental control of chaos in plasmas has been published recently [14], where chaotic self-oscillations in the strong-current mode of a multidipole confined thermionic discharge have been tamed by a relatively weak periodic modulation of the discharge voltage. This communication presents experimental evidence for the next step in the challenging problem of the control of plasma chaos, the active feedback control of a chaotic plasma wave phenomenon. Naturally, such phenomena are strongly related to the control of spatiotemporal chaos [15]. The plasma waves considered here are propagating ionization waves in the positive column of a simple neon cold-cathode glow discharge. The external periodic modulation of the discharge current allows one to study the nonlinear dynamics of a two-frequency system, where one frequency is given by the single, propagating ionization wave and the second by the modulation signal. The modulation degree $m = \delta I_d(t)/I_d$ is typically chosen in the realm of a few percent. [I_d is the time-averaged discharge current and $\delta I_d(t)$ is the temporally alternating discharge current.] Experiments have demonstrated that above a certain threshold of m it is possible to observe secondary Hopf bifurcations [16], three-tori, and a transition to deterministic chaotic behavior [17] with a low-dimensional phase space attractor [18].

A conventional cold-cathode discharge tube (length 50 cm, diameter 2 cm) is operated with neon as filling gas at a pressure of $p = 220$ Pa. The discharge is operated at a current of around $I_d = 10 - 20$ mA where ionization waves (moving striations) propagate [19,20]. The discharge current is modulated sinusoidally using a voltage-controlled current source. The modulation degree m is the control parameter and the local axial electric field strength $E(z_0, t)$ at a particular axial position z_0 is the dynamical observable. The electric field is obtained from the difference in the floating potential of two small axially separated cylindrical probes (probe diameter 0.05 mm, probe separation 5 cm). Time series are recorded with a 12 bit transient digitizer. Power spectra are measured with a high-resolution 14 bit fast-Fourier-transform analyzer (frequency resolution better than 5 Hz). An advantage of periodically driven dynamical systems is that an equivalent of the Poincaré map can be obtained directly from the stroboscopic mapping [21]: the observable is recorded for time instants $E(z_0, t_0 + kT) = X_k$ $(k = 1, 2, \ldots)$, where $T = f_i^{-1}$ is the period duration of the (injected) driver frequency f_i, and t_0 corresponds to a constant phase angle ϕ_0 with respect to the driver signal. The nth return map is constructed by plotting X_k versus X_{n+k}.

The feedback algorithm to control the chaotic behavior is similar to the occasional proportional feedback (OPF) method developed by Hunt [7]. As suggested by

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OGY, the control signal is generated from the observation of fixed points in the return map. UPO's of period n are found by noting that the corresponding (unknown) fixed point $X_{F,n}$ must lie along the $X_k = X_{k+n}$ line in the plane of the return map. As pointed out recently by Bielawski *et al.* [11], it is sufficient to choose the correction of the control parameter proportional to the differences of sampled values $(X_{k+n} - X_k)$ instead of $(X_{k+n} - X_{F,n})$. A stability analysis essentially revealed that in this case stabilization of UPO's is possible for any unstable Floquet multiplier if feedback is not applied at each period. This allows one to stabilize the UPO even when the control parameter is swept in a wide range. Based on these results, we have experimentally obtained the control signal as follows. The observable X_k is sampled at time instants $t_0 + kT$. The X_k values are stored for n driver periods using an analog sample-and-hold circuit while the succeeding X_{k+l} are recorded. Only if the condition $l = n$ is met, i.e., n driver periods have been passed, *and* if both X_k and X_{k+n} are found within a small, suitable window $[w_0, w_1]$, is the output of a difference amplifier $Z_k = \alpha(X_{k+n} - X_k)$ added to the driver signal (α is an adjustable feedback factor). In the following it is demonstrated that this approach makes it possible to stabilize UPO's in the chaotic state of the periodically driven glow discharge.

The discharge is operated at a discharge current of $I_0 = 13.9$ mA. Without any external modulation the oscillation frequency of the electric field is $f_0 = 2312$ Hz. The discharge is periodically driven with a frequency $f_i = 4450$ Hz, and above a critical modulation degree of $m_c = 5\%$ a direct transition to chaotic behavior occurs. In Fig. 1 for the chaotic state the time series of the local electric field fluctuations and its power spectrum $S(f)$ are shown. The correlation dimension of the corresponding chaotic phase space attractor was estimated to be $D_2 \approx 3.5$ [18] and the largest Lyapunov exponent is positive. The pronounced single peak in the power spectrum, Fig. 1(a), is the driver signal f_i whereas the spectral components in the regime $f < f_i$ are broadly distributed, as expected for a chaotic state. In Fig. 2 the effect of the feedback control to stabilize a period-3 orbit is shown. Figure 2(a) shows stroboscopically recorded time series of electric field fluctuations X_k and the difference signal $\delta X_k^3 = X_{k+3} - X_k$ as the system is switched from no control to control about the period-3 orbit. After the activation of the feedback control, the fixed points are approached exponentially and after approximately 30 periods (≈ 7 ms) the formerly unstable orbit with periodicity $P = 3$ is fully stabilized. By the chosen control scheme, an UPO embedded in the chaotic attractor has been stabilized. This becomes more evident from the inspection of Fig. 2(b), the first return map of both the chaotic state (gray dots — no control) and the stabilized periodic orbit (black dots — control). The fixed points are found in dense regions of the chaotic attractor. Consequently the stabilized trajectory of the periodic orbit is embedded in the chaotic attractor. In the third return map, Fig. 2(b), the fixed points of the stabilized periodic orbit lie exactly along the $X_n = X_{n+3}$ line.

With the same approach and increasing but still low level of control parameter perturbations, it is possible to perform the stabilization of UPO's with periodicity $P = 2, 4, 8, 16$, and 32. Only the inherent noise level of $\delta X_n / X_n \le 0.5\%$ obscures the detection of periodicities higher than $P = 32$. Even though period doubling bifurcations are rarely observed in the periodically driven neon glow discharge [22], this sequence of stabilized phase space orbits is preferred by the discharge system, that is, only small control parameter changes $\delta m < 5\%$ are necessary. In Fig. 3, the power spectrum, time series, and first return map of the stabilized period-16 orbit are shown. The control algorithm described above performs a correction of the control parameter m after each $n = 16$ full periods of the driver signal. The time series of the driver signal [Fig. 3(a)] makes evident the relatively weak modifications of the modulation degree. As a general rule, orbits of high periodicity need stronger perturbations of the control parameter. Note that the control signal is superimposed on a constant offset value. Consequently, the UPO is stabilized and the time series of the local electric field strength shows a strict periodicity of $P = 16$ [cf. Fig. 3(a)]. The time-averaged power spectrum [Fig. 3(b)] strongly differs from that of the previous chaotic state [cf. Fig. 1(a)] and reveals the strict periodicity of the observed signal. Peaked spectral components

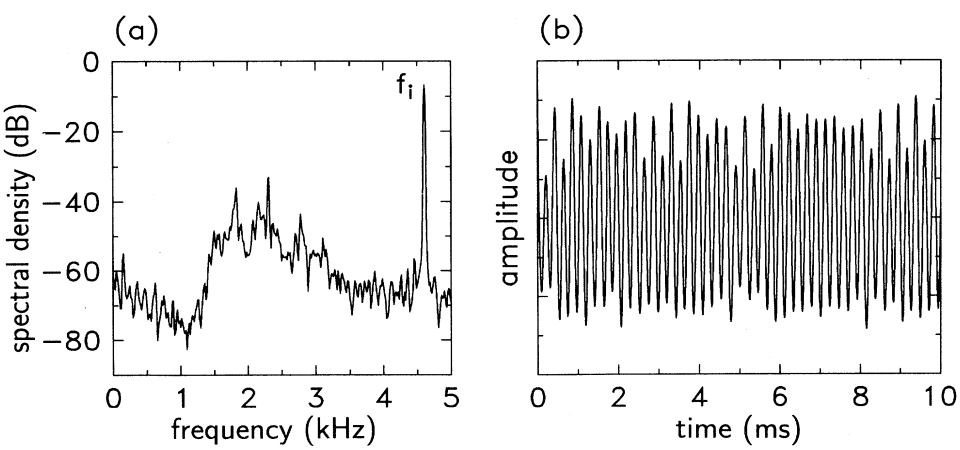

FIG. 1. Dynamical behavior of the perodically driven neon glow discharge. The driver frequency is $f_i = 4450$ Hz. The observable is the local electric field strength $E(z_0, t)$ at an axial position z_0 in the positive column. Shown are (a) the time-averaged power spectrum $S(f)$ and (b) the time series of the local electric field.

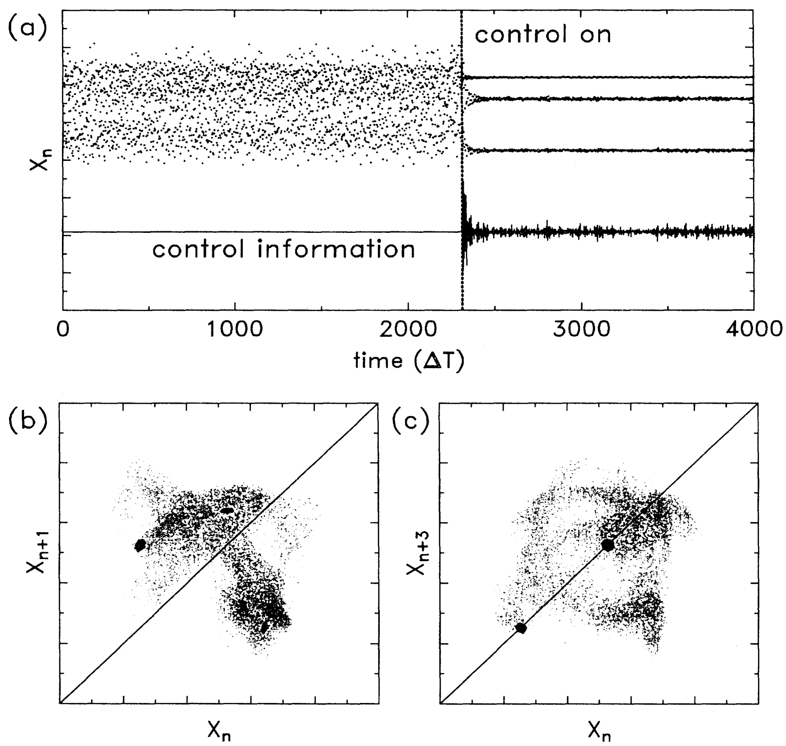

FIG. 2. (a) Stroboscopically recorded time series of the local electric field strength over a time period during which the control circuit is switched on. An unstable period-3 orbit is stabilized based on the control information shown in the bottom of the graph. (b) The first return map of the dynamics shows both the Poncaré section of the chaotic attractor (in gray) and the fixed points of the stabilized orbit. Clearly, the stabilized orbit is embedded into the chaotic attractor. (c) In the third return map, the fixed points lie along the $X_n = X_{n+3}$ line.

are found for integer multiples of $f_i/16$ only. The first return map of the stroboscopically recorded data [Fig. 3(c)] shows 16 distinct points. Again, the periodic orbits are embedded into the chaotic attractor, i.e., part of the (uncontrolled) chaotic dynamics. It should be emphasized that without active feedback such a dynamical state cannot be stabilized with the given values of f_0 and f_i. A simple experiment demonstrates the importance of an active control scheme for the stabilization of UPO's in the glow discharge: it is tried to suppress the chaos by periodic modulation of the control parameter (see above), i.e., by a simple periodic amplitude modulation (AM) of the driver signal. It turns out that a much higher AM (approximately a factor of 20) is necessary to induce a

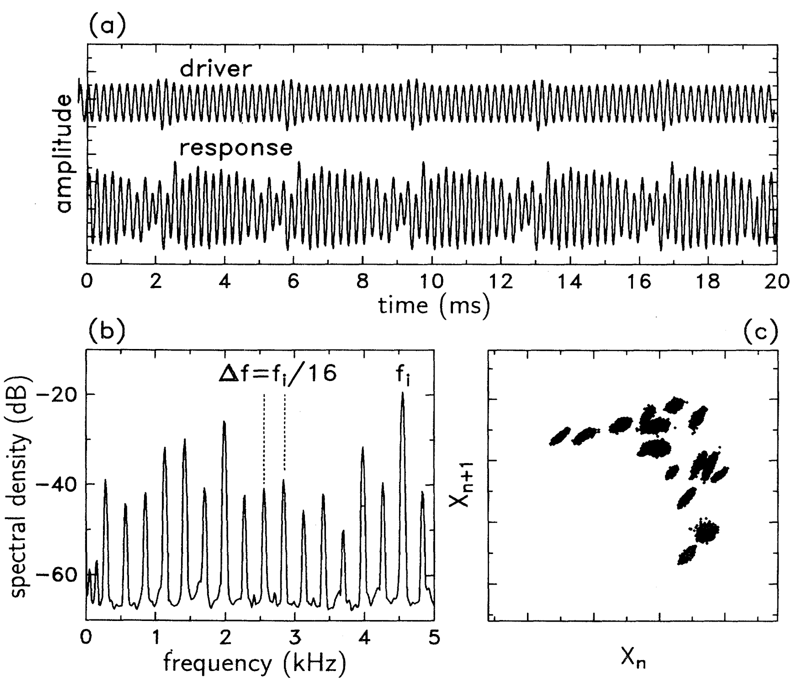

FIG. 3. Stabilization of a high-periodic orbit with periodicity $P = 16$. (a) Time series of the driver signal (top trace) and the local electric field fluctuations (bottom trace). Note the slight corrections of the driver amplitude after each 16 driver periods. (b) Power spectrum of the local electric field fluctuations. Spectral components are found at integer multiples of $f_i/16$ only. (c) First return map of the period-16 state.

periodic state. Hence only the active feedback control allows the suppression of chaos by small perturbations of the control parameter.

In conclusion, it was demonstrated that it is possible to stabilize unstable periodic orbits in the chaotic state of a periodically driven neon glow discharge by an active feedback technique, based on the algorithm suggested by Ott, Grebogy, and Yorke. Only small variations of the modulation amplitude are necessary for stabilization that turned out to be robust against noise. The observation of the control of a plasma wave phenomenon may also have impact on experimental support for spatiotemporal chaos. It has been pointed out recently by Aranson *et al.* [23] that the weak control of waves at a single point can suppress spatiotemporal chaos.

The authors wish to thank Professor H. Deutsch, Professor M. E. Koepke, and Professor A. Piel for their encouragement and helpful discussions. One author (T.K.) gratefully acknowledges the kind hospitality of the Greifswald plasma dynamics group. We would like to thank Th. Mausbach for expert experimental support in the early stage of the experiments. S. Gubsch is thanked for technical support. This work was performed under the auspices of the Sonderforschungsbereich 198 "kinetics of partially ionized plasmas" of the Deutsche Forschungsgemeinschaft, Projects No. A7 and No. A8.

[1] E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
[2] E. Ott, T. Sauer, and J. A. Yorke, *Coping with Chaos* (Wiley, New York, 1994).
[3] R. Lima and M. Pettini, Phys. Rev. A **41**, 726 (1990).
[4] L. Fronzoni, M. Giocondo, and M. Pettini, Phys. Rev. A **43**, 6483 (1991).
[5] Y. S. Kivshar, F. Rodelsperger, and H. Benner, Phys. Rev. E **49**, 319 (1994).
[6] Y. Braiman and I. Goldhirsch, Phys. Rev. Lett. **66**, 2545 (1991).
[7] E. R. Hunt, Phys. Rev. Lett. **67**, 1953 (1991).
[8] K. Pyragas and A. Tomaševičius, Phys. Lett. A **180**, 99 (1993).
[9] Z. Yu *et al.*, Phys. Rev. E **49**, 220 (1994).
[10] R. Roy, T. W. Murphy, T. D. Maier, Z. Gills, and E. R. Hunt, Phys. Rev. Lett. **68**, 1259 (1992)
[11] S. Bielawski, D. Derozier, and P. Glorieux, Phys. Rev. A **47**, 2492 (1993).
[12] B. Peng, V. Petrov, and K. Shovalter, J. Phys. Chem. **95**, 4957 (1991).
[13] A. K. Sen, Phys. Fluids B **5** 3997 (1993).
[14] W. X. Ding, H. Q. She, W. Huang, and C. X. Yu, Phys. Rev. Lett. **72**, 96 (1994).
[15] D. Auerbach, Phys. Rev. Lett. **72**, 1184 (1994).
[16] C. Wilke, R. W. Leven, and H. Deutsch, Phys. Lett. A **136**, 114 (1989).
[17] B. Albrecht, H. Deutsch, R. W. Leven, and C. Wilke, Phys. Scr. **47**, 196 (1993).
[18] K.-D. Weltmann, H. Deutsch, H. Unger, and C. Wilke, Contrib. Plasma Phys. **33**, 73 (1993).
[19] N. L. Oleson and A. W. Cooper, Adv. Electron. Electron Phys. **24**, 155 (1968).
[20] A. Rutscher, Habilitationsschrift, Ernst-Moritz-Arndt Universität Greifswald, 1964 (in German).
[21] E. A. Jackson, *Perspectives of Nonlinear Dynamics* (Cambridge University Press, Cambridge, England, 1991), Vol. 1.
[22] C. Wilke, H. Deutsch, and R. W. Leven, Contrib. Plasma Phys. **30**, 659 (1990).
[23] I. Aranson, H. Levine, and L. Tsimring, Phys. Rev. Lett. **72**, 2561 (1994).